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CROSS CORRELATION LOSS DUE TO BOTTOM REFLECTION.(U)

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U. S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

(6) CROSS CORRELATION LOSS DUE TO BOTTOM REFLECTION,

USL Problem No.
1-055-00-00

by

(10) Benjamin F. Cron

USL Technical Memorandum No. 913-54-65

(11) 7 April 1965

(14) USL-TM-913-54-65

INTRODUCTION

The bottom is considered to act as a linear filter. For specific conditions, the literature shows that this filter has a phase shift that is constant for all frequencies and an exponential attenuation with frequency. A random function that has a flat finite bandwidth is considered. The problem is to obtain the cross correlation for the direct with the bottom reflected version of this random function. A mathematical equation is derived. This equation is now being programmed. Curves are presented for the special case of phase shift only. Curves for the more general case will be presented at a later date.

(9) Technical memo.

THEORY

Consider a noise function $x(t)$ whose power spectrum $W_x(f)$ is flat in the frequency region f_1 to f_2. Let $\Delta f = f_2 - f_1$

(15) 11 p.

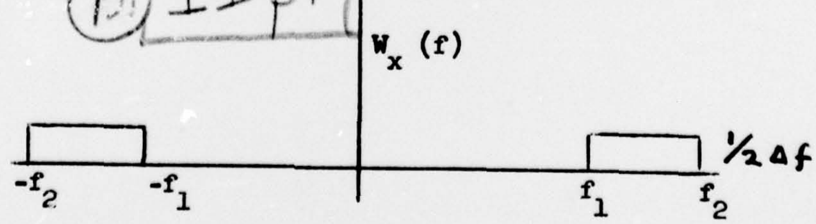

Figure 1

Let Φ (T) be the autocorrelation function of x(t). The autocorrelation function is the Fourier transform of the power spectrum.

Thus
$$\phi(\tau) = \int_{-\infty}^{\infty} W_x(f) \exp(i 2\pi f \tau) df \qquad (1)$$

$$\phi(\tau) = \mathrm{Re}\left\{ \int_{f_1}^{f_2} 2 \frac{1}{2\Delta f} \exp(i 2\pi f \tau) df \right\}$$

since $W_x(-f) = W_x(f)$

$$\phi(\tau) = \mathrm{Re}\left\{ \frac{\exp i 2\pi f_2 \tau - \exp i 2\pi f_1 \tau}{\Delta f \, i 2\pi \tau} \right\}$$

$$\phi(\tau) = \frac{\sin 2\pi f_2 \tau - \sin 2\pi f_1 \tau}{2\pi \Delta f \tau}$$

$$\phi(\tau) = \cos \omega_0 \tau \, \frac{\sin \pi \Delta f \tau}{\pi \Delta f \tau} \qquad (2)$$

Let us now pass this noise through a linear filter of transfer function H(f). Let y(t) be the output of the linear filter.

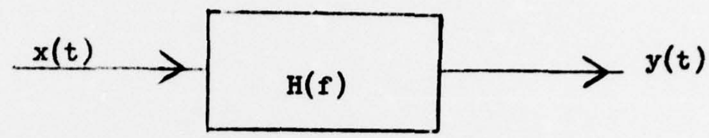

Figure 2

Let $W_{xy}(f)$ be the cross spectral density between the output and the input of the linear filter. It is well known that

$$W_{xy}(f) = H(f)\, W_x(f) \tag{3}$$

Let us now find the cross correlation of the output with the input of the filter. This cross correlation function is the Fourier transform of the cross spectral density. Thus

$$\phi_{xy}(\tau) = \int_{-\infty}^{\infty} W_{xy}(f) \exp(i\, 2\pi f \tau)\, df \tag{4}$$

$$\phi_{xy}(\tau) = \int_{-\infty}^{\infty} H(f)\, W_x(f) \exp(i\, 2\pi f \tau)\, df \tag{5}$$

$$\phi_{xy}(\tau) = \mathrm{Re}\left\{ \int_{0}^{\infty} 2\, H(f)\, W_x(f) \exp(i\, 2\pi f \tau)\, df \right\} \tag{6}$$

since $H(-f) = H^{*}(f)$

$W_x(-f) = W_x(f)$

For our case

$$\phi_{xy}(\tau) = Re\left\{ \int_{f_1}^{f_2} 2\, H(f)\, W_x(f) \exp(i 2\pi f \tau)\, df \right\} \tag{7}$$

Let $H(f) = \exp(i\epsilon) \exp(-2\pi b f)\,, \quad f > 0$

$\qquad = \exp(-i\epsilon) \exp(2\pi b f)\,, \quad f < 0$

This is the transfer function of a filter that has a constant phase shift for all frequencies and an exponential attenuation with frequency. For this case

$$\phi_{xy}(\tau) = Re\left\{ \int_{f_1}^{f_2} \frac{\exp(i\epsilon - 2\pi b f) \exp(i 2\pi f \tau)\, df}{\Delta f} \right\} \tag{8}$$

Performing the necessary algebraic manipulation, we obtain

$$\phi_{xy}(\tau) = \exp(-2\pi b f_0)\, Re\left\{ \frac{\exp[i(2\pi f_0 \tau + \epsilon)] \sin(\pi \Delta f(\tau + i b))}{\pi \Delta f(\tau + i b)} \right\} \tag{9}$$

For the special case of b = 0, we obtain

$$\phi_{xy}(\tau) = \cos(\omega_0 \tau + \epsilon) \frac{\sin \frac{\Delta \omega \tau}{2}}{\Delta \omega \tau / 2} \tag{10}$$

This is the case of constant phase shift but no attenuation. Equation (10) represents a result obtained by Wysor Marsh (reference (a)). Carrying out the algebra for equation (9), we obtain

$$\phi_{xy}(\tau) = \frac{e^{-2\pi b f_0}}{2\pi \Delta f \sqrt{\tau^2+b^2}} \Big\{ e^{\pi \Delta f b} \cos\left(2\pi f_1 \tau + \tan^{-1}\frac{\tau}{b} + \epsilon\right) \tag{11}$$

$$- e^{-\pi \Delta f b} \cos\left(2\pi f_2 \tau + \tan^{-1} \tau/b + \epsilon\right) \Big\}$$

We must now normalize this function. Let $\hat{\Phi}_{xy}(\tau)$ represent the normalized cross correlation function. By definition

$$\hat{\phi}_{xy}(\tau) = \frac{\phi_{xy}(\tau)}{\sqrt{\phi_x(0)\ \phi_y(0)}} \tag{12}$$

In this equation it is assumed that the average values of the input function and the output function are zero.

$$\phi_x(0) = \int_{-\infty}^{\infty} W_x(f)\, df = 1 \tag{13}$$

$$\phi_y(0) = \int_{-\infty}^{\infty} |H(f)|^2 W_x(f)\, df$$

$$\phi_y(0) = \frac{1}{\Delta f} \int_{f_1}^{f_2} \exp(-4\pi b f)\, df$$

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$$\phi_y(0) = \exp(-4\pi b f_0)\,\frac{\sinh(2\pi b\,\Delta f)}{(2\pi b\,\Delta f)} \tag{14}$$

Substituting equations (11), (13) and (14) into equation (12), we obtain

$$\hat{\phi}_{xy}(\tau) = \left\{ \frac{e^{\pi b \Delta f}\cos\left(2\pi f_1 \tau + \tan^{-1}\frac{\tau}{b} + \epsilon\right) - e^{-\pi b \Delta f}\cos\left(2\pi f_2 \tau + \tan^{-1}\frac{\tau}{b} + \epsilon\right)}{\sqrt{2\pi \Delta f b}\,\sqrt{1+\frac{\tau^2}{b^2}}\,\sqrt{\sinh(2\pi b \Delta f)}} \right\} \tag{15}$$

Equation (15) is the desired equation of the cross correlation of the output with the input.

BOTTOM REFLECTION

Consider a transmitter and a receiver in the ocean medium. Let there be two paths between the transmitter and receiver, namely the direct path and the bottom-reflected path.

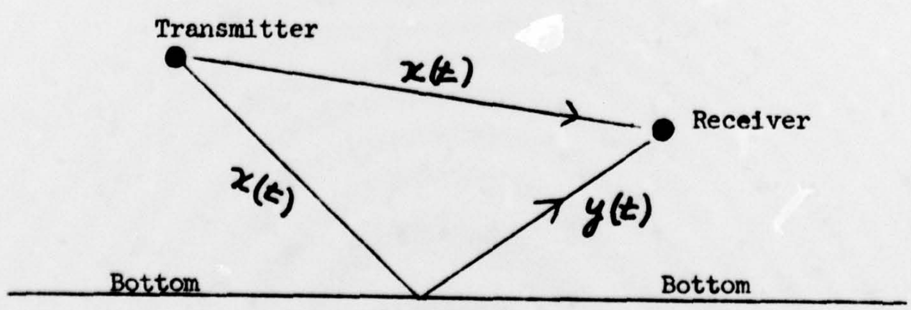

Figure 3

As shown by Cole (reference (b)), the bottom under certain conditions has a transfer function

$$H(f) = \exp(i\epsilon)\exp(-2\pi b f) \quad , \quad f > 0$$

Let us consider a random function x(t) chosen from an ensemble of functions. The average power spectrum of the ensemble is flat with frequency in the range of f_1 to f_2 . The function x(t) is of duration T such that the two paths can be resolved. Let y(t) be the reflected pulse. The problem is to find the correlation of the ensemble of x(t) with the corresponding y(t). The desired normalized cross correlation is given by equation (15).

A computer program is now being written for the evaluation of equation (15). Equation (15) will be evaluated for various parameters. Equation (10) is a special case of equation (15). This is the case of no attenuation and only a phase shift ϵ. This case has been also solved by Long (reference (c)). The cross correlation curves for $Q = 1/2$ and $Q = 1\ 1/2$, where $Q = f_o/\Delta f$ are given in Figures 4 and 5. For $Q = 1/2$ for a phase shift $\epsilon = \pi/2$, the maximum correlation peak is 0.7 instead of 1.0 For $Q = 1\ 1/2$, the maximum peak is very close to 1.0 so that the degradation due to phase shift is negligible for this case.

CONCLUSIONS

A mathematical equation for the cross correlation of a random function of flat bandwidth in a finite bandwidth with a phse shifted and exponentially attenuated function, has been derived. This cross correlation equation is being computed for various parameters. The case of no attenuation and only phase shift has been computed. It has been shown that for this case there is a negligible loss in cross correlation gain, when $Q > 1.5$

Benjamin F. Cron
BENJAMIN F. CRON
Research Associate

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ADMINISTRATIVE INFORMATION

This work is part of a joint effort between the author and A. H. Nuttall of Litton Systems Inc.

LIST OF REFERENCES

(a) H. W. Marsh, "Correlation in Wave Fields", USL Tech Memo 921-54-51 dtd 24 Jul 1951

(b) B. F. Cole, "Marine Sediment Attenuation and Ocean Bottom Reflected Sound" Article submitted to the Journal of the Acoustical Society of America

(c) E. Long, Admiralty Research Lab. No. ARL/N.59/L, USL Accession No. 37864, dtd June 1962, LIMITED CIRCULATION ONLY, CONFIDENTIAL

CROSS-CORRELATION LOSS
DUE TO PHASE DISTORTION

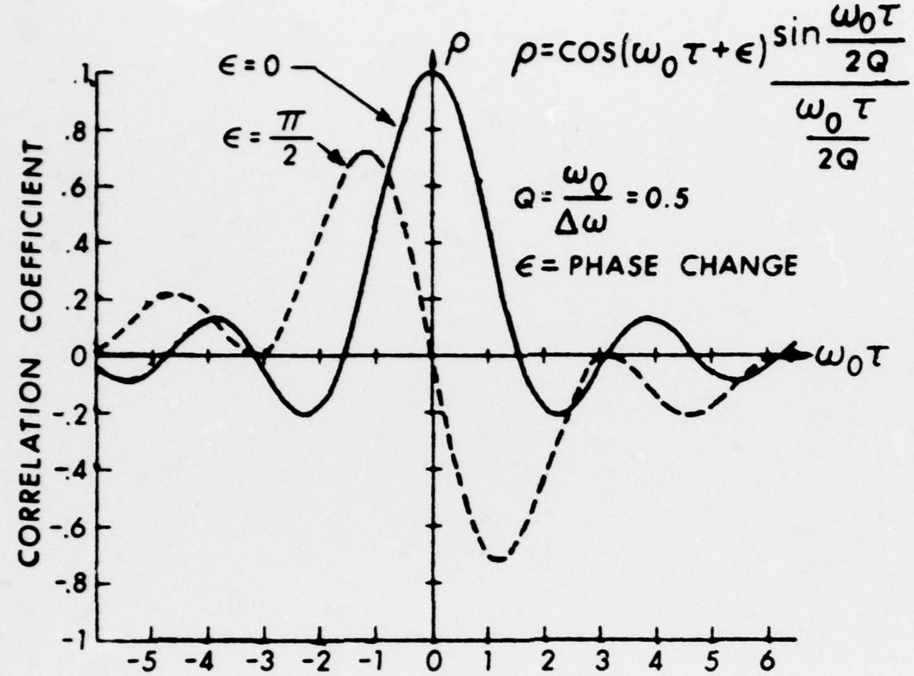

Figure 4
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CROSS—CORRELATION LOSS DUE TO PHASE DISTORTION

$$\rho = \cos(\omega_0\tau + \epsilon)\,\frac{\sin\frac{\omega_0\tau}{2Q}}{\frac{\omega_0\tau}{2Q}}$$

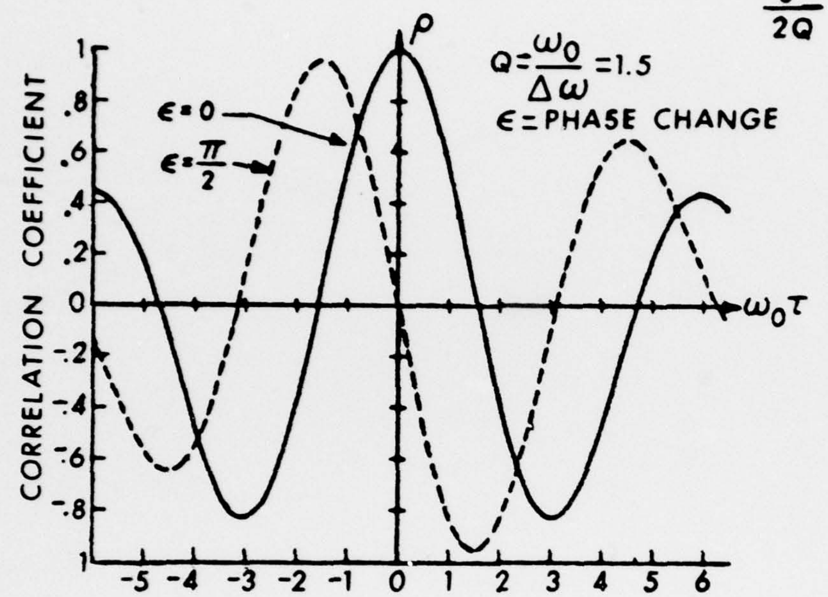

Figure 5
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